B.Sc. (Honours) Examination, 2018
Semester-III
Statistics
Course : CC-5

(Sampling Distribution)

Time : 3 Hours **Full Marks : 40**

Questions are of value as indicated in the margin

Answer **any four** questions

1. a) Let X_1, X_2,…,X_n be a sequence of i.i.d. random variables with common p.d.f. $f(x)=\begin{cases} e^{-(x-\theta)} & ; \quad x \ge \theta \\ 0 & ; \quad 0.\omega \end{cases}$

   Show that $\bar{X}_n \xrightarrow{P} 1+\theta$, where $\bar{X}_n = \dfrac{X_1+X_2+...+X_n}{n}$

   b) Explain why a random sample of size 25 to be preferred to a random sample of size 20 to estimate population mean.

   c) Show that if the sample is drawn without replacement then $\text{cov}(X_i, X_j) = \dfrac{-\sigma^2}{N-1}$, N being the population size and σ^2 population variance. 5+2+3=10

2. a) Define p-value in the context of testing of hypothesis.

   A student wishes to test a null hypothesis to calculate a probability of .65, but then realizes it is a two-failed test and he doubles this to get a p-value as 1.3. Comment on the validity of this.

   b) Show that t distribution tends to normality as $n \to \infty$.

   c) For X and Y ; independently distributed random variable each in the form R(0,1). Show that $U_1 = \sqrt{-2\ln X_1}\cos 2\pi X_2$ and $U_2\sqrt{-2\ln X_1}\sin 2\pi X_2$ are independently distributed N(0,1). 3+3+4=10

3. a) For the following sequence of independent random variables, does the weak law of large number hold?

   $$P\{X_k = \pm k\} = \frac{1}{2\sqrt{k}} \text{ and } P\{X_k = 0\} = 1 - \frac{1}{\sqrt{k}}.$$

   b) State and prove the Chebychev's inequality. 5+5=10

4. a) Describe a test procedure of difference of two sample proportions.

   b) Let X_1 and X_2 be normal variables with zero means, unit variances and correlation coefficient ρ, prove that

   $$E(\max(X_1, X_2)) = \sqrt{\frac{1-\rho}{\pi}}.$$ 5+5=10

P.T.O.

5. Let $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ be the set of order statistics of independent random variables X_1, $X_2, \ldots, X_n$ with common p.d.f.

$$f(x) = \begin{cases} \beta e^{-x/\beta} & if \quad X \geq 0 \\ 0 & \text{o.w} \end{cases}$$

a) Show that $X_{(r)}$ and $X_{(s)} - X_{(r)}$ are independently distributed for s<r.

b) Find the p.d.f. of $X_{(r+1)} - X_{(r)}$ and $E(X_{(r+1)} - X_{(r)})$. 5+5=10

6. a) Show that sample mean and sample variance are independently distributed when a sample is collected from $N(\mu, \sigma^2)$. Also establish the sampling distribution of sample variance.

b) Let the variables X_i (i = 1,2,3) be independently and identically distributed in the form N(0,1). Express the $P[4X_1^2 + 4X_2^2 - 7X_3^2 \leq 0]$ in terms of probability of χ^2 or t or F. 5+5=10